Exam. Code : 211002
Subject Code : 4901

M.Sc. (Mathematics) 2nd Semester (Batch 2021-23)

MATH-565 : PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt *five* questions in all, selecting at least *one* question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Formulate the PDE by eliminating the arbitrary function : $\Phi(x+y+z,\ x^2+y^2+z^2) = 0$. 5

(b) Find the integral surface of the linear PDE $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contain the straight line $x + y = 0$, $z = 1$. 5

(c) Solve the Cauchy problem for $(1+x^2)p + xyq = 0$, $x(0, y) = y^2$. 10

2. (a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2 = 1$, $z = 1$. 10

(b) Find the complete integral of $p^2 - y^2q = y^2 - x^2$. 5

(c) Solve $(3DD'-2D'^2-D')z = \sin(2x+3y)$. 5

SECTION—B

3. (a) Solve the given partial differential equation : $xs + q = 4x + 2y + 2$. 5

(b) Classify $u_{xx} + u_{yy} + u_{zz} + y_{yz} + u_{zy} = 0$. 5

(c) Reduce the equation $xys - x^2r - px - qy + z = -2xy^2z$ to canonical form and hence solve it. 10

4. (a) Solve the equation $r + s(a+b) + abt = xy$ by Monge's method. 10

(b) Find the solution of Laplace equation by using method of separation of variables

$$u_{xx} + u_{yy} = 0,\ 0 < x < 2,\ 0 < y < 2,$$

subjected to $u(x,0) = x^2$, $u(x, 2) = 0$, $u(0, y) = y$, $u(2, t) = 0$. 10

SECTION—C

5. (a) Find the resolvent kernel of the Volterra integral equation with the kernel : $K(x,t) = e^{x-t}$. 5

(b) Solve the integral equation : $\sin x = \lambda \int_0^x e^{x-\xi} u(\xi)d\xi$. 5

(c) Solve $y(x) = 1 + \int_0^x y(t)dt$. 5

(d) Using method of successive approximations, solve the integral equation :

$$y(x) = x - \int_0^x (x-t)y(t)dt,\ y_0(x) = 0.$$ 5

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6. (a) Form the Volterra integral equation corresponding to equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0$, with initial conditions y(0) = 1 and $\frac{dy}{dx}(0) = 0$. 10

(b) Solve $y(x) = 29 + 6x + \int_0^x [5 - 6(x - t)] y(t)\, dt$. 10

SECTION—D

7. (a) Form the Fredholm integral equation corresponding to the BVP :

$$\frac{d^2y}{dx^2} = f(x),\ y(0) = 0,\ y(1) = 0.$$ 10

(b) Find the iterated kernel for the kernel $K(x, t) = e^x \cos t$ of $y(x) = f(x) + \lambda \int_0^\pi K(x,t) y(t)\, dt$. 5

(c) Determine the resolvent kernel for the Fredholm integral equation having kernel :

$$K(x, t) = (1 + x)(1 - t);\ a = -1,\ b = 1.$$ 5

8. (a) Solve the given integral equation by the method of successive approximations :

$$y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\, y(t)dt.$$ 10

(b) Solve the following homogeneous equation :

$$y(x) = \frac{1}{e^2 - 1}\int_0^1 2e^x e^t y(t)dt.$$ 5

(c) Solve the Fredholm integral equation :

$$y(x) = 1 + \lambda \int_0^{\pi} \sin(x+t)y(t)dt.$$ 5